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Magneto-electro viscoelastic layer in functionally graded materials

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ABSTRACT

This paper deals with the problem of generalized magneto-thermoelasticity interactions in a functionally graded viscoelastic layer (Kelvin–Voigt type) due to the presence of thermal shock in the context of the linear theory of generalized thermoelasticity without energy dissipation (GN Model Type II). The partial differential equations governing stress, strain, displacement, temperature, induced electric field and induced magnetic field, functions are derived for one–dimensional layer problem of isotropic functionally graded magneto-electro-viscoelastic materials (FGMs). These equations are expressed in Laplace transform domain. The analytical solution in the transform domain is obtained by using a direct approach. The inversion of Laplace transform is done numerically. The numerical estimate of the temperature, displacement, strain, stress, induced electric field and induced magnetic field are obtained for a hypothetical material. The solution to the analogous problem for homogeneous isotropic material is obtained by taking non-homogeneity parameter suitably. Finally the results obtained are presented graphically to show the effect of non-homogeneity on the considered functions.

1. Introduction

Thermoelasticity theories which admit a finite speed for thermal signals (second sound) have aroused much interest in the last three decades.

These theories, known as generalized theories, involves hyperbolic type heat transport equation in contrast to the classical coupled thermoelasticity involving parabolic type (diffusion type) heat transport equation, which predicts infinite speed of propagation of thermal signals. Among the generalized theories the extended thermoelasticity proposed by Lord and Shulman [1] and the temperature rate dependent thermoelasticity proposed by Green and Lindsay [2] have been the subject of recent investigation. In view of experimental evidence in support of the finiteness of the speed of propagation of heat wave, generalized thermoelasticity theories are more acceptable than the conventional thermoelasticity theories in dealing with practical problems involving very short time intervals and high heat fluxes, like those occurring in laser units, energy channels and nuclear reactor.

Later Green and Naghdi [3] developed three models for generalized thermoelasticity of homogeneous and isotropic materials which are labeled as Models I, II, and III. The nature of these theories are such that when the respective theories are linearized, Model I reduced to the classical heat conduction theory (based on Fourier's law). The linearized versions of Models II and III permit propagation of thermal waves at finite speed. Model II, in particular, exhibits a feature that is not present in the other established thermoelastic models as it does not sustain dissipation of thermal energy [4,5]. In this model the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. The Green–Naghdi third model admits the dissipation of energy.

With the rapid development of polymer science and plastic industry, as well as the wide use of materials under high temperature in modern technology and application of biology and geology in engineering, the theoretical study and applications in viscoelastic material has become an important task for solid mechanics.

The theory of thermo-viscoelasticity and the solutions of some boundary value problem of thermoviscoelasticity were investigated by Ilioushin and Pobedria [6]. The works of Biot [7,8], Morland and Lee [9], Tanner [10], and Huilgol and Phan–Thien [11] made great strides in the last decade in finding solutions for boundary value problems for Linear viscoelasticity materials including temperature variations in both quasi static and dynamic problems. Drozdov [12] derived a constitutive model in thermoviscoelasticity which accounts for changes in elastic moduli and relaxation times.

The theory of electro-magneto-thermo-viscoelasticity has aroused much interest in many industrial applications, particularly in nuclear device, where there exists a primary magnetic field. Various investigations have been carried out by considering the interaction between magnetic, thermal and strain fields. Analyses of such problem also influence various applications in biomedical engineering as well as in different geometric studies. Misra et al. [13] have studied a one-dimensional uncoupled magnetic-thermoelastic problem in a viscoelastic medium using Maclaurin's

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Nomenclature

λ_e, μ_e	Lame's elastic constant	α_t	coefficient of linear thermal expansion
λ_v, μ_v	Lame's viscoelastic constant	γ	$=(3\lambda_e + 2\mu_e)\alpha_t$
ρ	density	E	electric displacement vector
C_E	specific heat at constant strain	J	current density vector
t	time	H	total magnetic intensity vector
θ	temperature increment	h	induced magnetic field vector
σ_{ij}	components of stress tensor	H_o	initial uniform magnetic field
e_{ij}	components of strain tensor	μ	magnetic permeability
U_i	components of displacement vector	δ_{ij}	Kronecker delta
K	thermal conductivity		

approximation method valid for only a specific range of parameters. Ezzat et al. [14,15] introduced the state-space approach for the model of two-dimension equations of generalized thermoviscoelasticity with one and two relaxation time.

Recently, various problems in solid mechanics are being studied where the elastic coefficients are no longer constant but they are functions of position. The investigations result from the fact that idea of non-homogeneity in elastic coefficients is not at all hypothetical, but more realistic. Elastic properties in soil may vary considerably with positions. The earth crust it self is non-homogeneous. Beside these, some structural materials such as functionally graded materials (FGMs) have distinct non-homogeneous character. Functionally graded materials (FGMs) are a new class of advances composite materials wherein the composition of each material constituent varies gradually with respect to spatial coordinates. FGMs possess continuously and smoothly varying material properties and this distinguishes FGMs from the laminated composite materials in which the abrupt change in materials properties across the interface between layers can result in large interlaminar stresses leading to delimitation.

The materials are made to utilize desirable properties of their individual constituent. For example, thermal protection plate structures made of a two-phase ceramic/metal functionally graded (FG) composite provide heat and corrosion resistance on the ceramic-rich surface while maintaining the structural strength and stiffness by the metal-rich surface. The concept of FGM was proposed in 1984 by a group of materials scientists, in Sendai, Japan, for thermal barrier or heat shielding properties [16–18]. These types of inhomogeneous composite materials and systems are presently in the forefront of materials research receiving worldwide attention and much research activities have been accelerated. The advantages of using these materials is that they are able to withstand high-temperature gradient environments while maintaining their structural integrity. The composition is varied from a ceramic-rich surface to metal-rich surface with a desired variation of the volume fraction of the two materials in between two surfaces can be easily manufactured [19]. Initially, FGMs were designed as thermal barrier materials for aerospace application and fusion reactors, later on, FGMs are developed for military, automotive, biomedical application, semiconductor industry, manufacturing industry and general structural element in thermal environments [20]. FGMs allow for spatial optimization by grading the volume fractions of two or more constituents to improve the response of structures. If properly designed, FGMs can offer various advantages such as reduction of thermal stresses, minimization of stress concentration or intensity factors and attenuation of stress waves. Hence, FGMs have gained potential applications in a wide variety of engineering components or systems which include the rocking-motor casing, armor plating, heat-engine components, packaging encapsulates, thermoelectric generators, and human implants. Several analytical solutions have been proposed for the thermoelastic deformation of structural components made of FGMs. Sugano [21] analyzed the one-dimensional transient thermal stress problem of non-homogeneous plate where the thermal conductivity and Young's modulus vary exponentially, whereas Poisson's ratio and the coefficient of linear thermal expansion vary arbitrary in the thickness direction. Jeon et al. [22] presented the analytical treatments for the steady thermoelastic problems of non-homogeneous slabs, assuming that the shear modulus of elasticity, the thermal conductivity and the coefficient of linear thermal expansion vary with the power product form of axial coordinate variable. Vel and Batra [23] and Qian and Batra [24] analyzed the three dimensional steady or transient thermal stress problems of functionally graded rectangular plate whose material properties vary with the power product from through the thickness. On the other hand, since shell type structures are used in various industrial fields, the thermoelastic analysis of circular cylinders, spheres and cylindrical panels made of FGM becomes important. Obata and Noda [25] analyzed the one-dimensional functionally graded hollow cylinder and hollow sphere using a perturbation method. Lutz and Zimmerman [26] presented the exact solutions for one-dimensional thermal stresses of functionally graded sphere whose elastic modulus and coefficient of linear thermal expansion vary linearly with radius. Ye et al. [27] presented the exact solution for the axisymmetric thermoelastic problem of a uniformly heated functionally graded transversely isotropic cylindrical shell, assuming that the modulus of elasticity and the coefficient of linear thermal expansion vary with the power product form of radial coordinates variable. El-Naggar et al. [28] analyzed the transient thermal stresses in a rotating non-homogeneous orthotropic hollow cylinder using a finite difference method. Wang and Mai [29] analyzed the transient one-dimensional thermal stresses in non-homogeneous materials such as plates, cylinders, and spheres using a finite element method. Ootao and Tanigawa [30] studied exactly a one-dimensional transient thermoelastic problem of a functionally graded hollow cylinder whose thermal and thermoelastic constants are assumed to vary with the power product form of radial coordinate variable. Shao et al. [31] solved a thermo mechanical problem of a FGM hollow circular cylinder whose material properties are assumed to be temperature independent and vary continuously in the radial direction. Mallik et al. [32] solved a thermoelastic FGM with a periodically varying heat source in the context of linear theory of generalized thermoelasticity without energy dissipation (GN Model Type II).

The main object of the present work is to consider a one-dimensional magneto-thermoelastic disturbance in isotropic functionally graded viscoelastic medium in the context of generalized thermoelasticity without energy dissipation (GN Model Type II) in presence of thermal shock. The material properties of the FGM is assumed to be vary exponential with space variable. The governing equations for this problem are taken into Laplace transform domain. The solution for temperature, displacement, strain, stress, induced electric field and induced magnetic field in Laplace transform domain is obtained by using a direct approach. Then the inversion of Laplace transform have been carried out numerically by applying a method of numerical inversion of Laplace transform based on Fourier series expansion technique [33]. Numerical results for all variables in physical space–time domain have been obtained for a copper like material and have been presented graphically to show the effect of nonhomogeneity. It is observed that the results of associated homogeneous case may easily be recovered from our results by letting the nonhomogeneity parameter become zero.

2. Formulation of the problem

We shall consider a functionally graded isotropic thermo-viscoelastic body at a uniform reference temperature θ_o of a perfect electrically conductivity permeated by an initial magnetic field $\boldsymbol{H}_o$.

Due to the effect of this magnetic field there arises in the conducting medium an induced magnetic field $\boldsymbol{h}$ and induced electric field $\boldsymbol{E}$. Also, there arises a force $\boldsymbol{F}$ (The Lorentz Force).

We shall make two important restrictions, first, that the medium under consideration is a perfect electric conductor and second, that the initial magnetic field vector $\boldsymbol{H}_o$ is oriented in such a way that the propagation of plane waves in the xy-plane is possible.

The linearized equations of electromagnetism for slowly moving media are

$$curl\ \boldsymbol{h} = \boldsymbol{J}. \tag{1}$$

$$curl\ \boldsymbol{E} = -\mu\frac{\partial \boldsymbol{h}}{\partial t}. \tag{2}$$

$$\boldsymbol{B} = \mu\boldsymbol{H}. \tag{3}$$

$$div\ \boldsymbol{B} = 0. \tag{4}$$

The above field equations are supplemented by constitutive equations which consists of First, Ohm's law for perfect conducting medium [34]

$$\boldsymbol{E} = -\mu\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{t}} \times H. \tag{5}$$

Second, Lorentz force

$$\boldsymbol{F} = \boldsymbol{J} \times \boldsymbol{B}. \tag{6}$$

We shall consider one-dimensional disturbance of the medium, so that the displacement vector u and temperature field θ can be expressed in the following form:

$$\boldsymbol{u} = (u(x,t), 0, 0). \tag{7}$$

$$\theta = \theta(x,t). \tag{8}$$

Now, assume that the initial magnetic field H_o acts in the z-direction and has the components $(0, 0, H_o)$. The induced magnetic field h will have one component h in the z-direction, while the induced electric field E will have one component E in the y-direction.

Then, Eqs. 1, 2, 5 yield

$$J = H_o\frac{\partial e}{\partial x}. \tag{9}$$

$$h = -H_o e. \tag{10}$$

$$E = \mu H_o\frac{\partial u}{\partial t}. \tag{11}$$

From Eqs. (9) and (6), we get that Lorentz force has only one component F_x in the x-direction.

$$F_x = \mu H_o^2\frac{\partial e}{\partial x}. \tag{12}$$

The stress-displacement–temperature relation for viscoelastic medium of Kelvin–Voigt type [35].

$$\sigma_{ij} = 2\left(\mu_e + \mu_v\frac{\partial}{\partial t}\right)e_{ij} + \left(\lambda_e + \lambda_v\frac{\partial}{\partial t}\right)e\delta_{ij} - \gamma\theta\delta_{ij}. \tag{13}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), e = e_{ii}. \tag{14}$$

Stress equation of motion:

$$\rho\ddot{u}_i = \sigma_{ij,j} + F_i. \tag{15}$$

Heat equation corresponding to generalized thermoelasticity without energy dissipation is

$$K\theta_{,ii} = \rho c_E\ddot{\theta} + \gamma\theta_o\ddot{e}. \tag{16}$$

With the effects of functionally graded solid, the parameters $\lambda_e, \lambda_v, \mu_e, \mu_v, \mu, k, \gamma$ and ρ no longer constant but become space-dependent. Thus we replace $\lambda_e, \lambda_v, \mu_e, \mu_v, \mu, k, \gamma$ and ρ by:

$$\lambda_e^o f(\boldsymbol{x}), \lambda_v^o f(\boldsymbol{x}), \mu_e^o f(\boldsymbol{x}), \mu_v^o f(\boldsymbol{x}), \mu^o f(\boldsymbol{x}), K^o f(\boldsymbol{x}), \gamma^o f(\boldsymbol{x}) \text{ and } \rho^o f$$

respectively, where $\lambda_e^o, \lambda_v^o, \mu_e^o, \mu_v^o, \mu^o, K^o, \gamma^o$ and ρ^o are assumed to be constants and $f(\boldsymbol{x})$ is a given non-dimensional function of space variable

$x = x(x, y, z)$.

Then Eqs. (9)–(11), (13), (15), (and) (16) take the following form:

$$J = H_o\frac{\partial e}{\partial x}. \tag{17}$$

$$H = -H_o e. \tag{18}$$

$$E = \mu^o H_o f(\boldsymbol{x})\frac{\partial u}{\partial t}. \tag{19}$$

$$\sigma_{ij} = f(\boldsymbol{x})\left[2\left(\mu_e^o + \mu_v^o\frac{\partial}{\partial t}\right)e_{ij} + \left(\lambda_e^o + \lambda_v^o\frac{\partial}{\partial t}\right)e\delta_{ij} - \gamma^o\theta\delta_{ij}\right]. \tag{20}$$

$$f(\boldsymbol{x})\rho^o\ddot{u}_i = f(\boldsymbol{x})\left[2\left(\mu_e^o + \mu_v^o\frac{\partial}{\partial t}\right)e_{ij} + \left(\lambda_e^o + \lambda_v^o\frac{\partial}{\partial t}\right)e\delta_{ij} - \gamma^o\theta\delta_{ij}\right]_{,j}$$
$$+f(\boldsymbol{x})_{,j}\left[2\left(\mu_e^o + \mu_v^o\frac{\partial}{\partial t}\right)e_{ij} + \left(\lambda_e^o + \lambda_v^o\frac{\partial}{\partial t}\right)e\delta_{ij} - \gamma^o\theta\delta_{ij}\right] + \mu^o H_o^2 f(\boldsymbol{x})\frac{\partial e}{\partial x}. \tag{21}$$

$$(K^o f(\boldsymbol{x})\theta_{,i})_{,i} = \rho^o c_E f(\boldsymbol{x})\ddot{\theta} + \gamma^o\theta_o f(\boldsymbol{x})\ddot{e}. \tag{22}$$

It is assumed that material properties depend only on the x co-ordinate. So, we can take $f(\boldsymbol{x})$ as $f(\boldsymbol{x})$.

In the context of linear theory of generalized thermoelasticity based on Green–Naghdi Model II. Eqs. (17)–(22) can be written as:

$$J = H_o\frac{\partial e}{\partial x}. \tag{23}$$

$$h = -H_o e. \tag{24}$$

$$E = \mu^o H_o f(x)\frac{\partial u}{\partial t}. \tag{25}$$

$$\sigma_{xx} = f(x)\left[\left[(\lambda_e^o + 2\mu_e^o) + (\lambda_e^o + 2\mu_e^o)\frac{\partial}{\partial t}\right]\frac{\partial u}{\partial x} - \gamma^o\theta\right]. \tag{26}$$

$$\rho^o f(x)\tfrac{\partial^2 u}{\partial t^2} = f(x)\left[\left[(\lambda_e^o + 2\mu_e^o) + (\lambda_v^o + 2\mu_v^o)\tfrac{\partial}{\partial t} + \mu^o H_o^2\right]\tfrac{\partial u}{\partial x} - \gamma^o\tfrac{\partial\theta}{\partial x}\right]$$
$$+\left[\left[(\lambda_e^o + 2\mu_e^o) + (\lambda_v^o + 2\mu_v^o)\tfrac{\partial}{\partial t}\right]\tfrac{\partial u}{\partial x} - \gamma^o\theta\right]\tfrac{\partial f(x)}{\partial x}. \tag{27}$$

$$\frac{\partial}{\partial x}\left[K^o f(x)\frac{\partial\theta}{\partial x}\right] = \rho^o c_E f(x)\ddot{\theta} + \gamma^o\theta_o f(x)\ddot{e}. \tag{28}$$

where

$$e_{xx} = \frac{\partial u}{\partial x}. \tag{29}$$

Introducing the following non-dimensional variables.

$$\begin{gathered} x' = \tfrac{x}{L}, t' = \tfrac{c}{L}t, \theta' = \tfrac{\theta}{\theta_o}, \sigma'_{xx} = \tfrac{\sigma_{xx}}{\gamma^o\theta_o}, u' = \tfrac{\lambda_o+2\mu_o}{L\gamma^o\theta_o}u, e'_{xx} = e_{xx}, f(x') = f(x), \\ f(x') = f(x), h' = \tfrac{h}{H_o}, e'_{xx} = e_{xx}, f(x') = f(x), \mu_v^{o'} = \tfrac{L}{c}\mu_v^o, \lambda_v^{o'} = \tfrac{L}{c}\lambda_v^o, \\ E' = \tfrac{E}{\mu^o H_o c}, J' = \tfrac{L}{H_o}J. \end{gathered} \tag{30}$$

where L is a standard length and c is a standard speed, and omitting the primes Eqs. (23)–(29) can be re-written in non-dimensional form as:

$$J = \frac{\partial e}{\partial x}. \tag{31}$$

$$h = -e \tag{32}$$

$$E = f(x)\frac{\partial u}{\partial t} \tag{33}$$

$$\sigma_{xx} = f(x)\left[\left(1+\varepsilon\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x} - \theta\right]. \tag{34}$$

$$f(x)\frac{\partial^2 u}{\partial t^2} = f(x)\left[\left(1+\varepsilon\frac{\partial}{\partial t}+R_H\right)\frac{\partial^2 u}{\partial x^2} - \frac{\partial\theta}{\partial x}\right] + \left[\left(1+\varepsilon\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x} - \theta\right]\frac{\partial f}{\partial x}. \tag{35}$$

$$\varepsilon_1\frac{\partial}{\partial x}\left(f(x)\frac{\partial\theta}{\partial x}\right) = f(x)\frac{\partial^2\theta}{\partial t^2} + \varepsilon_2 f(x)\frac{\partial^3 u}{\sigma x\partial t^2}. \tag{36}$$

$$e_{xx} = \frac{\partial u}{\partial x}. \tag{37}$$

where

$$\varepsilon = \frac{\lambda_v^o + 2\mu_v^o}{\lambda_e^o + 2\mu_e^o}, \varepsilon_1 = \frac{K^o}{\rho^o c_E c^2}, \varepsilon_2 = \frac{\gamma^{o^2}\theta_o}{(\lambda_e^o + 2\mu_e^o)\rho^o c_E}, R_H = \frac{\mu^o H_o^2}{\lambda_e^o + 2\mu_e^o}.$$

The coefficient R_H represents the effect of the applied magnetic field on the thermoelastic process proceeding in the body.We assume that the medium is initially at rest. The undisturbed state is maintained at reference temperature. Then we have:

$$u(x,o) = \dot{u}(x,o) = \theta(x,o) = \dot{\theta}(x,o) = o. \tag{38}$$

3. Exponential variation of nonhomogeneity

We take $f(x) = e^{-\alpha x}$, where k is a dimensionless constant [32]. Then Eqs. (31)–(37) reduce to

$$J = \frac{\partial e}{\partial x}. \tag{39}$$

$$h = -e. \tag{40}$$

$$E = e^{-\alpha x}\frac{\partial u}{\partial t}. \tag{41}$$

$$\sigma_{xx} = e^{-\alpha x}\left[\left(1+\varepsilon\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x} - \theta\right]. \tag{42}$$

$$\frac{\partial^2 u}{\partial t^2} = \left(1+\varepsilon\frac{\partial}{\partial t}+R_H\right)\frac{\partial^2 u}{\partial x^2} - \frac{\partial\theta}{\partial x} - \alpha\left[\left(1+\varepsilon\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x} - \theta\right]. \tag{43}$$

$$\varepsilon_1\left(\frac{\partial^2\theta}{\partial x^2} - \alpha\frac{\partial\theta}{\partial x}\right) = \frac{\partial^2\theta}{\partial t^2} + \varepsilon_2\frac{\partial^3 u}{\partial x\partial t^2}. \tag{44}$$

$$e_{xx} = \frac{\partial u}{\partial x}. \tag{45}$$

4. Solution in the Laplace transform domain

Taking the Laplace transform defined by the relation.

$$\bar{f}(x,p) = L\{f(x,t)\} = \int_o^\infty e^{-pt} f(x,p)dt$$

on both side of Eqs. (39)–(45), we get:

$$\bar{J} = D\bar{e} \tag{46}$$

$$\bar{h} = -\bar{e}. \tag{47}$$

$$\bar{E} = e^{-\alpha x}p\bar{u}. \tag{48}$$

$$\bar{\sigma}_{xx} = e^{-\alpha x}[(1+\varepsilon p)D\bar{u} - \bar{\theta}]. \tag{49}$$

$$p^2\bar{u} = (1+\varepsilon p+R_H)D^2\bar{u} - D\bar{\theta} - \alpha[(1+\varepsilon p)D\bar{u} - \bar{\theta}], \tag{50}$$

$$\varepsilon_1(D^2\theta - \alpha D\bar{\theta}) = p^2\bar{\theta} + \varepsilon_2 p^2 D\bar{u}. \tag{51}$$

$$\bar{e}_{xx} = D\bar{u}. \tag{52}$$

where $D = \frac{d}{dx}$.

Eliminating $\bar{u}$ between Eqs. (49) and (50), we obtain the following fourth order equation satisfied by $\bar{\theta}$:

$$(D^4 + a_1 D^3 + a_2 D^2 + a_3 D + a_4)\bar{\theta} = 0. \tag{53}$$

where

$$a_1 = -[2\varepsilon\alpha(1+\varepsilon p) - \varepsilon_1\alpha R_H]/[\varepsilon_1(1+\varepsilon p+R_H)]$$

$$a_2 = [\varepsilon_1\alpha^2(1+\varepsilon p) - \varepsilon_1 p^2(1+\varepsilon p+R_H) - \varepsilon_2 p^2]/[\varepsilon_1(1+\varepsilon p+R_H)].$$

$$a_3 = [\alpha p^2(\varepsilon p + \varepsilon_1 + \varepsilon_2 + 1)]/[\varepsilon_1(1+\varepsilon_p+R_H)]$$

$$a_4 = p^4/[\varepsilon_1(1+\varepsilon_p+R_H)].$$

In a similar manner we can show that $\bar{u}$ satisfy the equation

$$(D^4 + a_1 D^3 + a_2 D^2 + a_3 D + a_4)\bar{u} = 0. \tag{54}$$

Now, we define $m_i, i = 1,2,3,4$ as the roots of the characteristic equation

$$m^4 + a_1 m^3 + a_2 m^2 + a_3 m + a_4 = 0. \tag{55}$$

The solution of Eqs. (53) and (54) can be written as:

$$\bar{\theta} = \sum_{i=1}^4 A_i e^{m_i x}. \tag{56}$$

$$\bar{u} = \sum_{i=1}^4 B_i e^{m_i x}. \tag{57}$$

where A_i and B_i are constants which can be determined from boundary conditions.

From Eq. (50) we get:

$$B_i = \frac{\varepsilon_1 m_1^2 - \varepsilon_1 \alpha m_i - p^2}{\varepsilon_2 m_i p^2} A_i, \quad i = 1, 2.3, 4. \tag{58}$$

From Eqs (56)–(58), (and) (49) we get:

$$\bar{\sigma}_{xx} = e^{-\alpha x} \sum_{i=1}^{4} [(1 + \varepsilon p) m_i B_i - A_i] e^{m_i x}. \tag{59}$$

Finally, we arrived at:

$$\bar{\theta} = \sum_{i=1}^{4} A_i e^{m_i x}. \tag{60}$$

$$\bar{u} = \sum_{i=1}^{4} \frac{\varepsilon_1 m_1^2 - \varepsilon_1 \alpha m_i - p^2}{\varepsilon_1 m_i p^2} A_i e^{m_i x}. \tag{61}$$

$$\bar{\sigma}_{xx} = e^{-\alpha x} \sum_{i=1}^{4} \frac{(1 + \varepsilon p)(\varepsilon_1 m_1^2 - \varepsilon_1 \alpha m_i - p^2) - \varepsilon_1 p^2}{\varepsilon_1 p^2} A_i e^{m_i x}. \tag{62}$$

$$\bar{e} = \sum_{i=1}^{4} \frac{\varepsilon_1 m_1^2 - \varepsilon_1 \alpha m_i - p^2}{\varepsilon_1 p^2} A_i e^{m_i x}. \tag{63}$$

$$\bar{J} = \sum_{i=1}^{4} \frac{m_i(\varepsilon_1 m_1^2 - \varepsilon_1 \alpha m_i - p^2)}{\varepsilon_1 p^2} A_i e^{m_i x}. \tag{64}$$

$$\bar{h} = -\sum_{i=1}^{4} \frac{\varepsilon_1 m_1^2 - \varepsilon_1 \alpha m_i - p^2}{\varepsilon_1 p^2} A_i e^{m_i x}. \tag{65}$$

$$\bar{E} = e^{-\alpha x} \sum_{i=1}^{4} \frac{(\varepsilon_1 m_1^2 - \varepsilon_1 \alpha m_i - p^2)}{\varepsilon_1 p^2} A_i e^{m_i x}. \tag{66}$$

5. Application

We consider the problem of a thick plate of finite high h. Choosing the x-axis perpendicular to the surface of the plate with the origin coinciding with the lower plate, the region Ω under consideration becomes:

$$\Omega = \{(x, y, z) : 0 < x < h, -\infty < y < \infty, -\infty < z < \infty\}.$$

The surface of the plate is taken to be traction free. The lower plate is subjected to a thermal shock $\theta = \theta_o H(t)$. The upper plate is kept at zero temperature.

The boundary conditions of the problem in the transformed domain are:

$$\bar{\theta}(o, p) = \frac{\theta_o}{p}. \tag{67}$$

$$\bar{\theta}(h, p) = o. \tag{68}$$

$$\bar{\sigma}_{xx}(o, p) = o. \tag{69}$$

$$\bar{\sigma}_{xx}(h, p) = o. \tag{70}$$

By substituting from Eqs. (67)–(70) into Eqs. (59) and (60) we get:

$$\sum_{i=1}^{4} A_i = \frac{\theta_o}{P}. \tag{71}$$

$$\sum_{i=1}^{4} A_i \overset{m_i h}{e} = o. \tag{72}$$

$$\sum_{i=1}^{4} \frac{(1 + \varepsilon p)(\varepsilon_1 m_i^2 - \varepsilon_1 m_j - p^2) - \varepsilon_1 p^2}{\varepsilon_1 p^2} A_i = o. \tag{73}$$

$$\sum_{i=1}^{4} \frac{(1 + \varepsilon p)(\varepsilon_1 m_i^2 - \varepsilon_1 m_j - p^2) - \varepsilon_1 p^2}{\varepsilon_1 p^2} A_i \overset{mih}{e} = o. \tag{74}$$

By solving this system of equations for $A_i (i = 1, 2, 3, 4)$ we can complete the solution of our problem.

6. Inversion of the Laplace transforms

In order to invert the Laplace transforms in the above equations we shall use a numerical technique based on Fourier expansions of functions.

Let $\bar{g}(s)$ be the Laplace transform of a given function $g(t)$. The inversion formula of Laplace transforms states that

$$g(t) = \frac{1}{2\pi i} \int_{d-i\infty}^{d+i\infty} e^{st} \bar{g}(s) ds$$

where d is an arbitrary positive constant greater than all the real parts of the singularities of $\bar{g}(s)$ .Taking $s = d + iy$, we get

$$g(t) = \frac{e^{dt}}{2\pi} \int_{-\infty}^{\infty} e^{ity} \bar{g}(d + iy) dy.$$

This integral can be approximated by

$$g(t) = \frac{e^{dt}}{2\pi} \sum_{k=-\infty}^{\infty} e^{ikt\Delta\Delta y} \bar{g}(d + ik\Delta y) \Delta y.$$

Taking $\Delta y = \frac{\pi}{t_1}$, we obtain:

$$g(t) = \frac{e^{dt}}{t_1} \left[\frac{1}{2} \bar{g}(d) + \text{Re} \left(\sum_{k=1}^{\infty} e^{ik\pi t/t_1} \bar{g}(d + ik\pi/t_1) \right) \right].$$

For numerical purposes this is approximated by the function

$$g_N(t) = \frac{e^{dt}}{t_1} \left[\frac{1}{2} \bar{g}(d) + \text{Re} \left(\sum_{k=1}^{N} e^{ik\pi t/t_1} \bar{g}(d + ik\pi/t_1) \right) \right]. \tag{75}$$

where N is a sufficiently large integer chosen such that

$$\frac{e^{dt}}{t_1} \text{Re}[e^{iN\pi t/t_1} \bar{g}(d + iN\pi/t_1)] < \eta$$

where η is a reselected small positive number that corresponds to the degree of accuracy to be achieved Formula (75) is the numerical inversion formula valid for $0 \leqslant t \leqslant 2t_1$ [33]. In particular, we choose $t = t_1$, getting

$$g_N(t) = \frac{e^{dt}}{t} \left[\frac{1}{2} \bar{g}(d) + \text{Re} \left(\sum_{k=1}^{N} (-1)^k \bar{g}(d + ik\pi/t) \right) \right]. \tag{76}$$

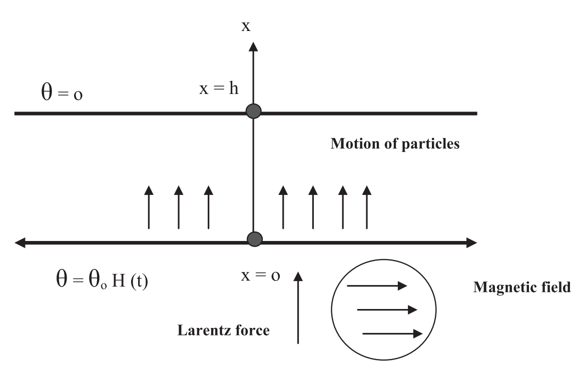

Fig. 1. The physical meaning of the problem.

7. Numerical results and discussion

To get the solution the thermal displacement, temperature, stress, strain, electric field, magnetic field and electric current in space time domain we have to apply Laplace inversion formula to the Eqs. (60)–(66). This has been done numerically using a method based on Fourier series expansion technique mentioned above. To get the roots of the characteristic equation (55) we have used DERIVE program. The numerical code has been prepared using Fortran 77 programming language. For the purpose of illustration we consider copper like material with material constants [36,37].

$$\varepsilon = 2, \varepsilon_2 = 0.0168, \mu_e = 0.448 \times 10^{12} \text{ dyne/cm}^2, \lambda_e = 1.387 \times 10^{12} \text{ dyne/cm}^2, \theta_o = 1^oC.$$

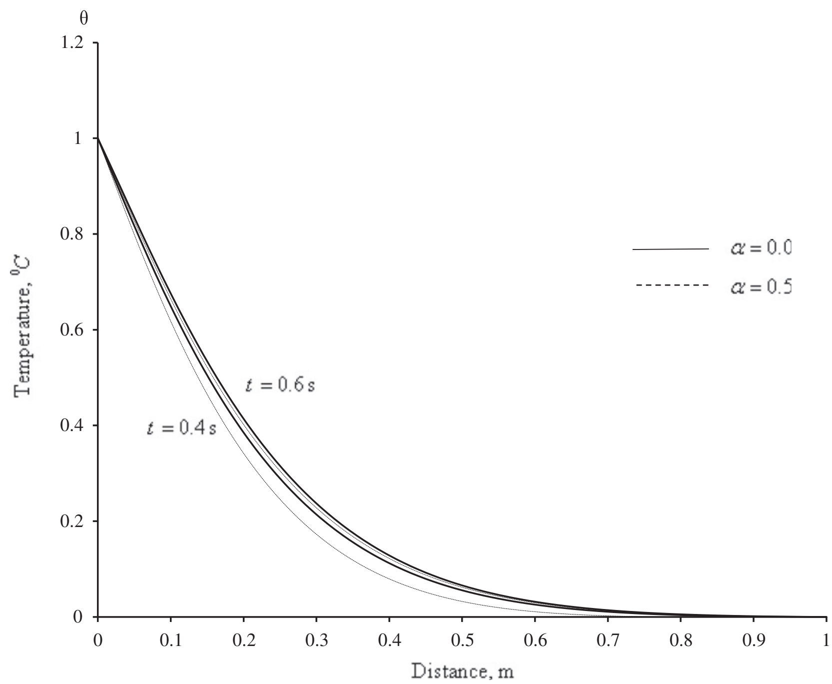

Fig. 2. Variation of temperature with distance for different values of α.

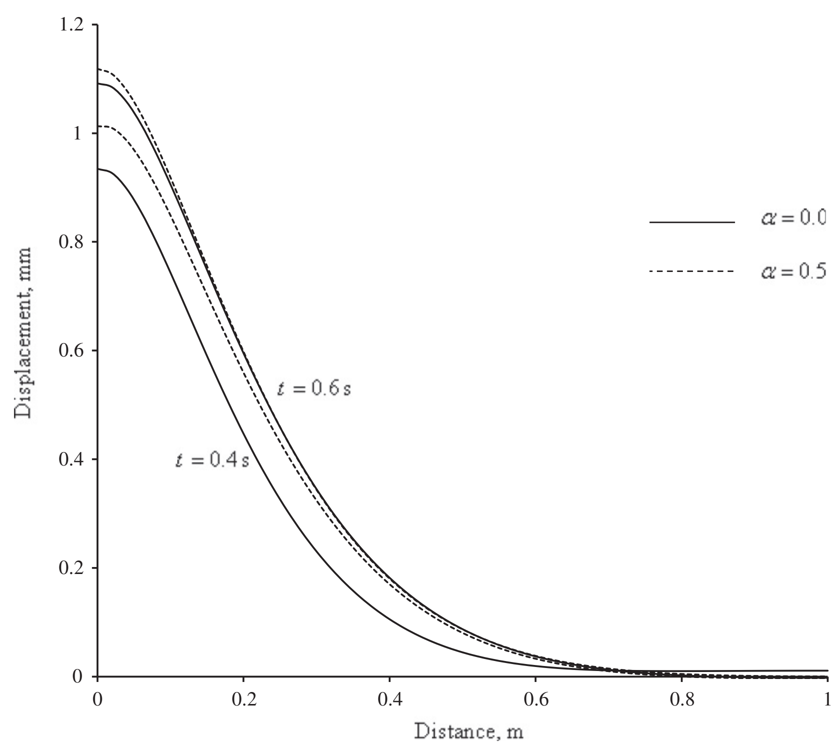

Fig. 3. Variation of displacement with distance for different values of α.

In order to study the effect of nonhomogeneity on thermal displacement, temperature, stress, strain, induced electric field and induced magnetic field we now present our results in the form of graphs (Figs. 1–9).

Fig. 2 is plotted to show the variation of temperature against x for $\alpha = 0$, 0.5 and $t = 0.4$, 0.6. It is observed from this figure as time t increase the magnitude of temperature increase, also we can notice that as the value of α increase the magnitude of the temperature decrease for fixed time $t = 0.4$ and ultimately θ approach to zero value.

Fig. 3 is plotted to show the variation of thermal displacement against x for $\alpha = 0$, 0.5 and $t = 0.4$, 0.6. It is observed from this figure as time t increase the magnitude of displacement increase, also we notice that as the value of the nonhomogeneity parameter α decreases the peak of the thermal displacement decreases.

Fig. 4 shows the variation of thermal displacement u against x for $t = 0.4$, $\alpha = 0.5$ and for different values of R_H. It is observed from this figure some difference in values of displacement is noticed for different value of the parameter R_H and on the lower plate, the values of displacement increase in the absence of magnetic field.

Fig. 5 is plotted to show variation of thermal stress versus distance x. Here the stress takes negative values for $\alpha = 0$, 0.5 and the magnitude of stress increases as α decrease.

Fig. 6 is plotted to show variation of thermal stress σ_{xx} versus distance x for $t = 0.4$, $\alpha = 0.5$ and for different values of R_H. Some

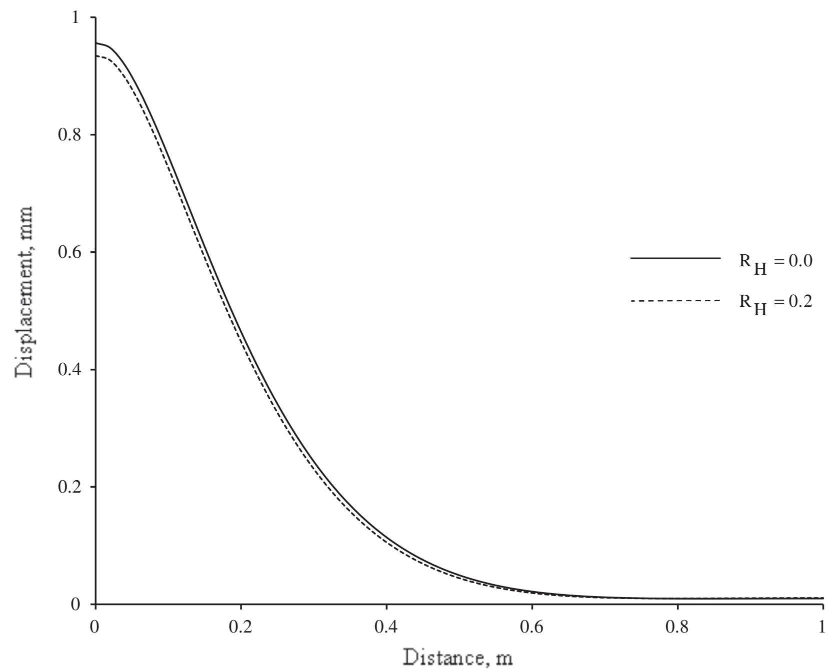

Fig. 4. Variation of displacement with distance for different values of R_H.

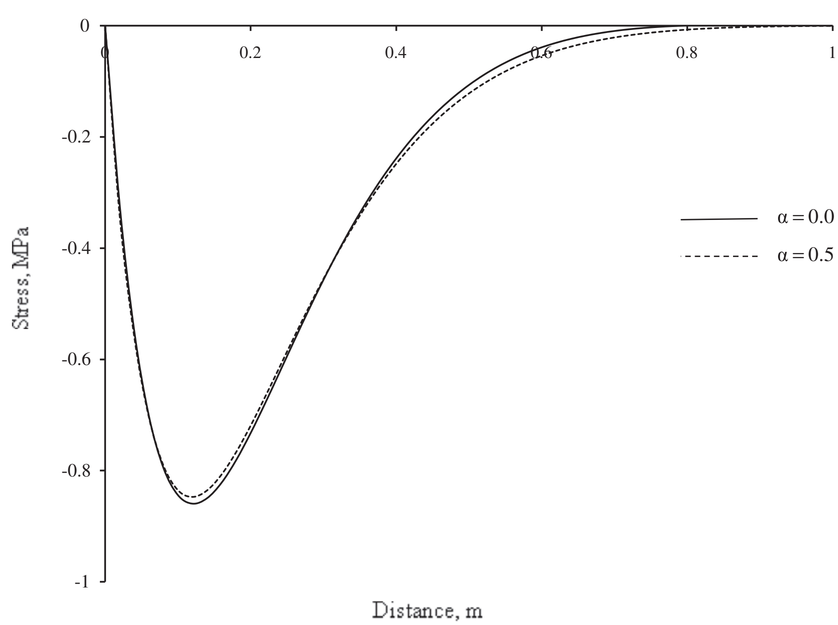

Fig. 5. Variation of stress with distance for different values of α.

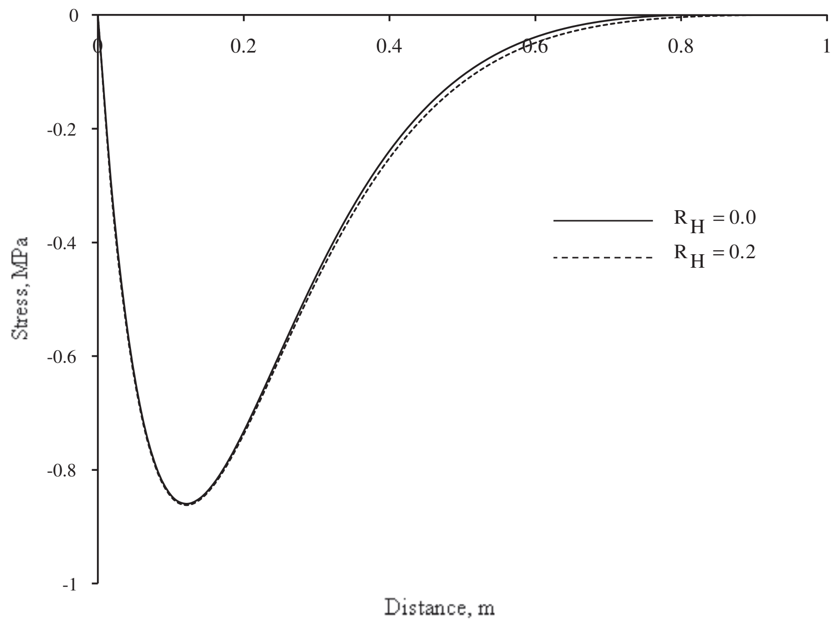

Fig. 6. Variation of stress with distance for different values of R_H.

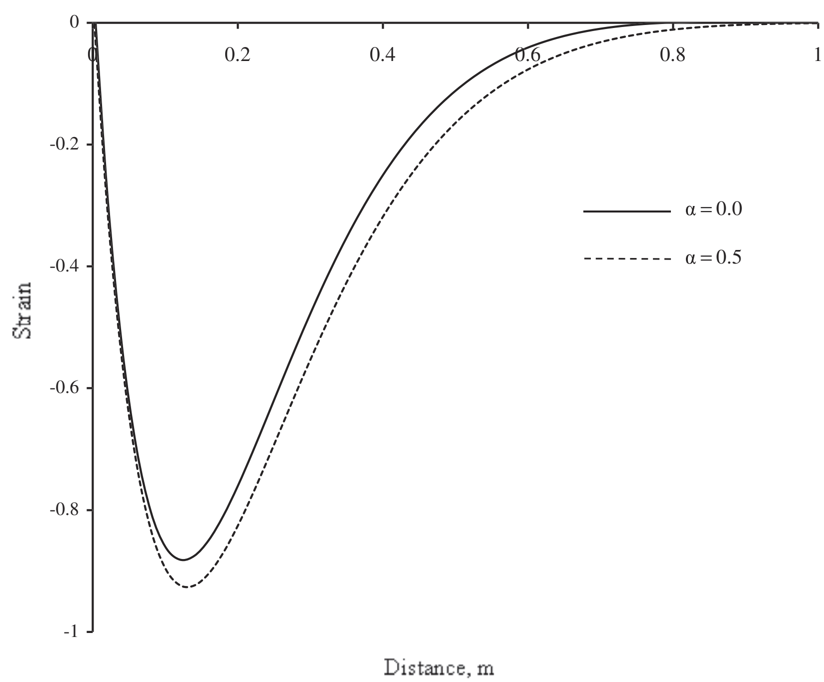

Fig. 7. Variation of strain with distance for different values of α.

difference in the values of the thermal stress is noticed for different values of R_H.

Fig. 7 gives the variation of thermal strain e against distance x. The strain takes positive values in the region $0 < x < 0.15$ ($\alpha = 0$), $0 < x < 0.01$ ($\alpha = 0.5$) and then negative value and finally diminishes to zero. The effect of nonhomogeneity is observed in the region $0.1 < x < 0.9$.

Fig. 8 gives the variation of thermal strain e against distance x for $t = 0.4$, $\alpha = 0.5$ and for different values of R_H. Some differences in the values of strain are noticed for different values of R_H. We can see that, the absolute value of the maximum point of the strain increase in the absence of magnetic field, (i.e., the magnetic field causes decreasing in the values of the strain).

Fig. 9 shows the variation of induced electric field E against distance x. The effect of nonhomogeneity is observed in the region $0.1 < x < 0.4$.

Fig. 10 shows the variation of induced magnetic field h against distance x .It is seen from the figure that as the value of α increase the magnitude of the induced magnetic field h also increase for fixed time $t = 0.4$ and ultimately h approach to zero value.

8. Conclusions

A methodology has been presented for the transient thermoelastic analysis of functionally graded perfect conducting materials

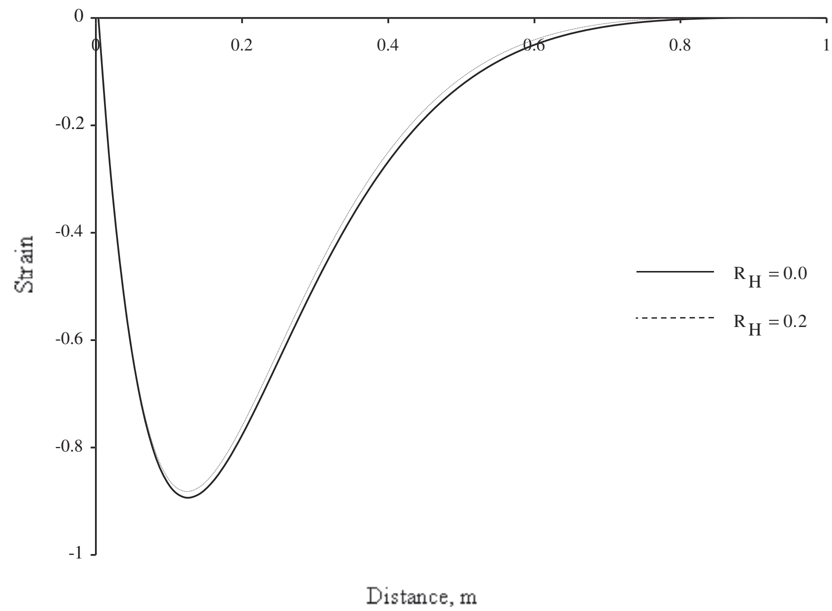

Fig. 8. Variation of strain with distance for different values of R_H.

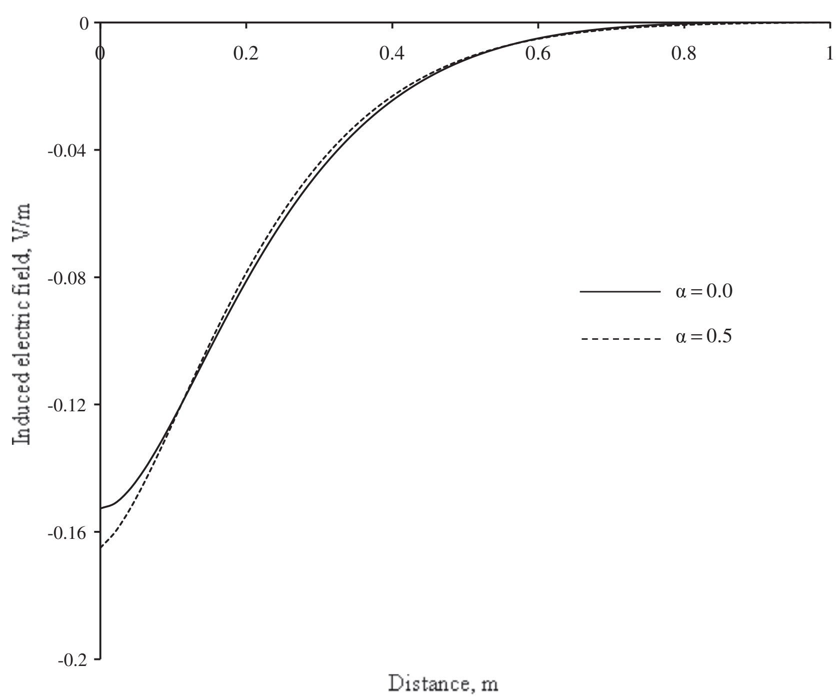

Fig. 9. Variation of induced electric field with distance for different values of α.

in the presence of a constant magnetic field. The presence of the nonhomogeneity parameter α has significant effect on the solutions of the non-dimensional temperature, displacement, stress, strain, induced electric field, and induced magnetic field distributions. It is observed that with the decrease of magnitude of the nonhomogeneity parameter the solution approaches to solution corresponding to the homogeneous problem and when the nonhomogeneity parameter $\alpha = 0$, the results tally in magnitude with the corresponding results of Chandrasekharaiah and Srinath [38] and Ezzat at el. [39].

The important phenomenon observed in the problem under concideration, where the medium is of infinite extent is that the solution of any of the considered function for the GN theories vanishes identically outside a bounded region of space. This demonstrates clearly the difference between the coupled and generalized theories of thermoelasticity. In the first and older theory the waves propagate with infinite speeds, so the value of any of the functions is not identically zero (though it may be very small) for any large value of x. In the generalized theory the response to the thermal effects does not reach infinity instantaneously but remains in a bounded region of space that expands with the passing of time [40].

The non-dimensional coefficient of the magnetic field R_H acts to decrease the displacement, the strain and the magnitude of the stress component. This is mainly due to the fact that the magnetic field corresponds to term signifying positive force that tends to

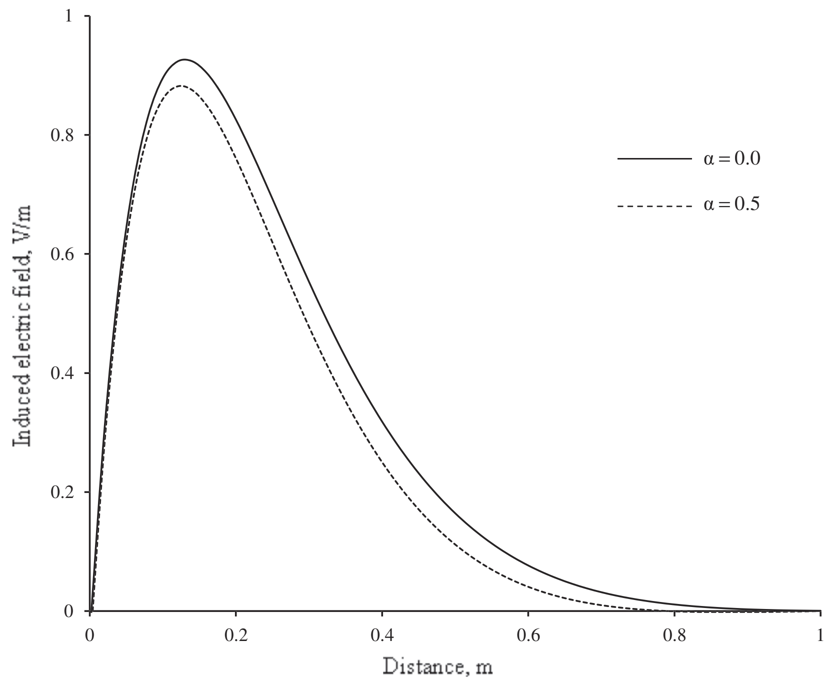

Fig. 10. Variation of induced magnetic field with distance for different values of α.

accelerate the charge carriers [41] and when $R_H = 0$ indicates the situation of the absence of magnetic field [32,42].

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